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LETTER TO THE EDITOR

On the paper by R J M Carr: 'Derivation of energy lower bound models for translation-invariant many-fermion systems'

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A detailed derivation of the energy lower bound model (SHRIMP model) for a translation-invariant many-fermion system has recently been proposed (Carr 1978). The author asserts that an N-particle shell model retaining antisymmetry in all N particles is obtained.

Here we want to point out the error in this derivation which makes the SHRIMP model wrong leaving correct the related RIP and HIP models.

The mistake is in the formula for energy (Carr's notation throughout):

$$E_0=\frac{1}{N}\sum_{i=1}^{N}(\Psi_0, \mathscr{H}_i(\boldsymbol{a}_i)\Psi_0)$$

$$=\frac{1}{N}\Big(\Psi_0, \Big(\sum_{j\neq 1} h_j(\boldsymbol{a}_1)+\sum_{j\neq 2} h_j(\boldsymbol{a}_2)+\cdots+\sum_{j\neq N} h_j(\boldsymbol{a}_N)\Big)\Psi_0\Big). \qquad (1)$$

Here the first equality is right, but the second equality does not hold. This becomes evident if one writes out explicitly coordinates of the wavefunction Ψ_0. Let us use the fact that Ψ_0 can be expressed solely in terms of the relative coordinates $\boldsymbol{\rho}_j=\boldsymbol{r}_j-\boldsymbol{r}_i (j\neq i)$, where $\boldsymbol{r}_i=\boldsymbol{a}_i$ (constant) as $m_i\to\infty$. We have:

$$\Psi_0(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N)=\phi_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \ldots, \boldsymbol{\rho}_{i-1}, \boldsymbol{\rho}_{i+1}, \ldots, \boldsymbol{\rho}_N)\equiv\phi_0[\boldsymbol{\rho}_j(j\neq i)]$$

and

$$E_0=\frac{1}{N}\sum_{i=1}^{N}\Big(\phi_0[(\boldsymbol{r}_j-\boldsymbol{a}_i)_{j\neq i}], \sum_{\substack{j=1\\ j\neq i}}^{N} h_j(\boldsymbol{a}_i)\phi_0[(\boldsymbol{r}_j-\boldsymbol{a}_i)_{j\neq i}]\Big).$$

It is seen, that ϕ_0 depends on i and cannot be taken out of the sum. This proves the statement that the second part of equation (1) is incorrect.

Taking into account that $h_j(\boldsymbol{a}_i)=f(\boldsymbol{\rho}_j)$ we get

$$E_0=\frac{1}{N}\int\cdots\int\phi_0^*(\boldsymbol{\rho}_2, \boldsymbol{\rho}_3, \ldots, \boldsymbol{\rho}_N)\sum_{j\neq 1} f(\boldsymbol{\rho}_j)\phi_0(\boldsymbol{\rho}_2, \boldsymbol{\rho}_3, \ldots, \boldsymbol{\rho}_N)\,\mathrm{d}\boldsymbol{\rho}_2\,\mathrm{d}\boldsymbol{\rho}_3\ldots\mathrm{d}\boldsymbol{\rho}_N$$

$$+\frac{1}{N}\int\cdots\int\phi_0^*(\boldsymbol{\rho}_1, \boldsymbol{\rho}_3, \ldots, \boldsymbol{\rho}_N)\sum_{j\neq 2} f(\boldsymbol{\rho}_j)\phi_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_3, \ldots, \boldsymbol{\rho}_N)$$

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$$\times \mathrm{d}\boldsymbol{\rho}_1\, \mathrm{d}\boldsymbol{\rho}_3 \ldots \mathrm{d}\boldsymbol{\rho}_N + \cdots$$
$$+\frac{1}{N}\int \cdots \int \phi_0^*(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \ldots, \boldsymbol{\rho}_{N-1}) \sum_{j \neq N} f(\boldsymbol{\rho}_j)\phi_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \ldots, \boldsymbol{\rho}_{N-1})$$
$$\times \mathrm{d}\boldsymbol{\rho}_1\, \mathrm{d}\boldsymbol{\rho}_2 \ldots \mathrm{d}\boldsymbol{\rho}_{N-1}.$$

It is evident that all these integrals differ only by the names of the integration variables and thus they all are equal. Hence

$$E_0 = \left(\phi_0[(\boldsymbol{r}_j - \boldsymbol{a}_1)_{j\neq 1}], \sum_{j=2}^{N} h_j(\boldsymbol{a}_1)\phi_0[(\boldsymbol{r}_j - \boldsymbol{a}_1)_{j\neq 1}]\right)$$

and we return to the RIP or HIP model.

References

Carr R J M 1978 *J. Phys. A: Math. Gen.* **11** 291